Proceedings of ASME 2017 Dynamic Systems and Control Conference
DSCC2017
October 11-13, 2017, Tysons, VA, USA

DSCC2017-5116

JUSTIFYING THE STABILIZATION OF A MARGINALLY STABLE SHIP

David Shekhtman
Department of Mechanical Engineering
The Cooper Union
New York, NY 10003
Email: shekht@cooper.edu

Dirk M. Luchtenburg
Department of Mechanical Engineering
The Cooper Union
New York, NY 10003
Email: dluchten@cooper.edu

ABSTRACT

As a precursor to capsize, marginal stability, resulting from incorrect loading conditions and crew negligence, poses a serious danger to ships. Therefore, as a benchmark problem for preventing capsize, the use of an actively controlled pendulum for the stabilization of a marginally stable ship was analyzed. Lyapunov stability criteria and closed loop eigenvalues were used to evaluate the extent to which a proposed pendulum controller could cope with different ship stability conditions. Equations of motion were solved to observe the controller's performance under different damping conditions. The behavior of the controller yielded the following results: a marginally stable ship can be stabilized, as long as there is no right hand plane zero; energy dissipation is key to the stabilization of a marginally stable ship; the controller must have knowledge of the ship's stability to prevent controller-induced excitation; and a stabilized tilted ship is more robust to external disturbances than a stabilized upright ship.

Nomenclature

m	Ship Mass including Pendulum Mass
I_{xx}	Ship Rotational Inertia
m_{cw}	Pendulum mass
g	Gravitational Acceleration
ϕ	Roll Angle
$\dot{\phi}$	Roll Anglular Velocity of Ship
θ_{cw}	Pendulum Angle
$\dot{\theta}_{cw}$	Pendulum Angular Velocity
$\theta_{cw,ref}$	Reference Pendulum Angle
b_{roll}	Ship Roll Damping
b_{pend}	Pendulum Damping
T_{pend}	Torque on the Pendulum
$T_{pend,eq}$	Pendulum Torque that Satisfies Equilibirum
z_g	z-coordinate of the Center of Mass (COM)
ρ_w	Density of Displaced Fluid
V	Ship Displaced Volume
$\overline{B_oG}$	Distance btw. Centers of Mass and Buoyancy at $\phi = 0^\circ$
$\overline{B_oM_x}$	Distance btw. Buoyancy Center and Metacenter
$\overline{GM_x}$	Metacentric Height
$\overline{GZ}$	Righting Arm
L_{cw}	Length of Pendulum
L_{off}	Pendulum Offset
k_1	Nonlinear Feedforward Reference Gain
k_2	Linear Feedback Gain
k_p	Proportional Feedback Gain
z_{ship}	Potential Right Half Plane Zero of Ship
p_{ship}	Right Half Plane Pole of Ship
E_{mech}	Total Mechanical Energy
KE	Total Kinetic Energy
PE	Total Potential Energy

1 INTRODUCTION

There are three approaches to preventing ship capsize: roll damping, dynamic vibration absorbing, and roll stabilization. Roll damping and vibration absorbing are the industry standards since they require the least actuation effort, considering the fact that a ship is designed to have a stable upright orientation. Chen explored nonlinearly controlled anti-roll tanks used as a dynamic vibration absorber to prevent boat capsize [1], and Pesman presented the use of roll damping to prevent ship capsize via the dissipation of mechanical energy [2]. Unfortunately, the most tragic and historic capsizes have occurred because of crew negligence, flooding, and incorrect loading [3]. In these situations, capsize is the result of a negative metacenter that forms a right half plane (RHP) pole, which indicates roll instability. Active roll stabilization for such a ship requires large forces and moments that can exceed the material limits of the ship and actuators. This is why conventional ship designs have low centers of mass. Yet, marginal roll stability exists in nature and can happen upon an unsuspecting vessel, the 2014 MV Sewol capsize being one tragic example [3], which calls for an active control scheme that can deal with marginal stability. Stabilization at marginal stability may be possible as long as required actuator effort and bandwidth are satisfied. This paper wishes to answer the question of whether a marginally stable ship can and should be actively stabilized.

Since ships are designed to be upright in calm seas, a stable but incorrectly loaded ship must pass through a point of marginal stability prior to entering a region of roll instability [4]. The regime near the marginal stability point should support active roll stabilization, until loading conditions can be corrected. The size of this stabilization regime depends on the speed of the pendulum and the energy dissipation rate due to roll damping. In technical terms, the roll actuation bandwidth of the ship is bounded by a RHP-zero due to roll damping in the upper limit and the RHP-pole due to a negative metacenter in the lower limit. Skogestad describes that the actuator bandwidth should be at least twice greater than the RHP-pole and at most half of that of the RHP-zero [5]. Care should be taken to avoid situations in which the RHP-zero is less than four times the RHP-pole—a case in which the ship can be only stabilized theoretically [5].

A ship can be viewed as a pendulum with a ball joint attached to the center of buoyancy [6]. With a center of mass below a critical height, a ship is self-righting and, in control terms, self-regulating. With a center of mass above the critical height, the boat will turn over like an inverted pendulum—this is one form of capsize. Here, the critical height represents a cross-over point, a point of marginal stability. The ship discussed in this paper is unlike a conventional ship. It is equipped with an actuated pendulum that shifts the ship's center of mass side to side, effectively transforming the overall ship system into a double pendulum. In order to determine whether the pendulum can stabilize the marginally stable ship, one turns to the equations of motion, Lyapunov stability analysis, and closed loop root locus plots, while at the same time keeping in mind Skogestad's warning.

2 CONTROLLING A MARGINALLY STABLE SHIP

In this section, the Lagrangian of the coupled ship-pendulum sytem is presented, followed by equations of motion and a discussion on static roll stability in the marginally stable regime. Then an actuator control law is proposed to stabilize a marginally stable ship and analyzed for its actuation effort and Lyapunov stability. Conclusions are also made about the practicality of controlling a marginally stable ship, based on parameter analyses.

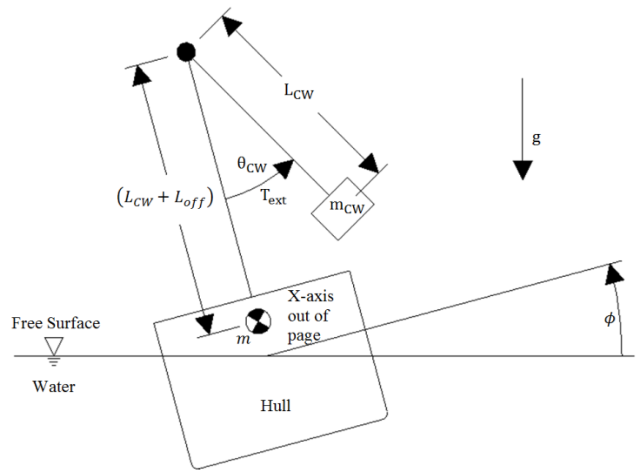

Figure 1: Sketch of the Ship-Pendulum Problem Setup.

2.1 Modeling of a ship

For simplicity, only the roll degree of freedom of a ship is considered. A sketch of our setup is shown in Fig. 1. The ship has a mass m and mass moment of inertia I_{xx}. For active roll control, a servo-actuated simple pendulum of length L_{cw} and mass m_{cw} is mounted at a height $L_{cw} + L_{off}$ about the center of mass of the ship. Let ϕ denote the roll angle and θ_{cw} the pendulum angle. Let g be the gravitational constant. For the derivation of the equations of motion EOM for the ship-pendulum system, differentiation is performed on the following Lagrangian:

$$L = KE - PE, \tag{1}$$

where the kinetic energy KE and potential energy PE are respectively

$$\begin{aligned} KE = \frac{\left(I_{xx} + m_{cw}(L_{cw} + L_{off})^2\right)\dot{\phi}^2}{2} + \frac{m_{cw}\dot{\theta}_{cw}^2 L_{cw}^2}{2} + \\ - m_{cw}\dot{\phi}\dot{\theta}_{cw}L_{cw}(L_{cw} + L_{off})\cos(\theta_{cw}) \end{aligned} \tag{2}$$

and

$$PE = mg\left(\overline{GM_x}(1-\cos(\phi)) - 0.5\overline{B_oM_x}(2-\sec(\phi)-\cos(\phi))\right) + m_{cw}g\left((L_{off}+L_{cw})\cos(\phi) - L_{cw}\cos(\phi+\theta_{cw})\right). \tag{3}$$

Nonconservative forces are added in afterwards, namely damping and torque input. In eq. (4), allow constant roll damping b_{roll} for the ship and constant viscous bearing damping b_{pend} for the pendulum servo. Let the servo apply an external torque T_{pend} on the pendulum. Note that the model neglects changes in ship buoyancy and servo start-up friction in the pendulum. If more degrees of freedom are desired, see Appendix A for a 7-DOF Lagrangian. In descriptor state space form, the EOM are given by

$$E\dot{\mathbf{x}} = f(\mathbf{x},\mathbf{u}), \tag{4}$$

where the state is $\mathbf{x} = \left[\phi\ \theta_{cw}\ \dot{\phi}\ \dot{\theta}_{cw}\right]^T$; the input is $\mathbf{u} = T_{pend}$; the coupling matrix is

$$\begin{aligned} E &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & e_1 & e_2 \\ 0 & 0 & e_2 & e_3 \end{bmatrix}, \\ e_1 &= I_{xx} + m_{cw}(L_{cw}+L_{off})^2 \\ e_2 &= -m_{cw}L_{cw}(L_{cw}+L_{off})\cos(\theta_{cw}) \\ e_3 &= m_{cw}L_{cw}^2; \end{aligned} \tag{5}$$

and the vector field is

$$\begin{aligned} \mathbf{f}(\mathbf{x},\mathbf{u}) &= \left[\dot{\phi}\ \dot{\theta}_{cw}\ f_1\ f_2\right]^T, \\ f_1 &= -m_{cw}\dot{\theta}_{cw}^2 L_{cw}(L_{cw}+L_{off})\sin(\theta_{cw}) + \\ &\quad -mg\sin(\phi)\left(\overline{GM_x} + \frac{\overline{B_oM_x}}{2}\tan^2(\phi)\right) - b_{roll}\dot{\phi} + \\ &\quad m_{cw}g\left((L_{cw}+L_{off})\sin(\phi) - L_{cw}\sin(\phi+\theta_{cw})\right) \\ f_2 &= -m_{cw}gL_{cw}\sin(\theta_{cw}+\phi) - b_{pend}\dot{\theta}_{cw} + T_{pend}, \end{aligned} \tag{6}$$

See Appendix B for the numerical values of the ship-pendulum system discussed in this paper. Note that the roll damping of the ship b_{roll} was estimated via the Ikeda method [2].

2.2 Marginal Stability and Justification for Control

The ship's equations of motion incorporate the classic wall-sided formula [7], which models the righting arm $\overline{GZ}$ of the buoyancy-gravity force couple for roll angles whose respective waterplanes do not exceed the walls of the ship (or else a piecewise treatment of the formula is required for higher roll angle intervals). A more dynamic alternative to the wall-sided formula, which incorporates ocean waves, can be found in [8], but by assuming calm, level seas, the wall-sided formula will suffice. Fig. 2 can be used to reconstruct the wall-sided formula, where point M is the Metacenter; point G is the center of mass COM; point B_o is the center of buoyancy at zero roll; and B is the current center of buoyancy of the ship at the roll angle ϕ. For a conventional ship, the wall-sided formula defines the static equilibrium roll angles of the ship and determines whether the ship is stable. For the actively controlled ship featured in this paper, the wall-sided formula describes static equilibrium roll angles if the pendulum angle is fixed, i.e. the pendulum is a dead load on the ship. Otherwise, for a freely rotating pendulum, the entire descriptor state space eq. (4) should be set to zero and solved for sets of equilibrium roll and pendulum angles. The wall-sided formula accounts for the ship's geometry and mass in a liquid of density ρ_w. Its expression is given by

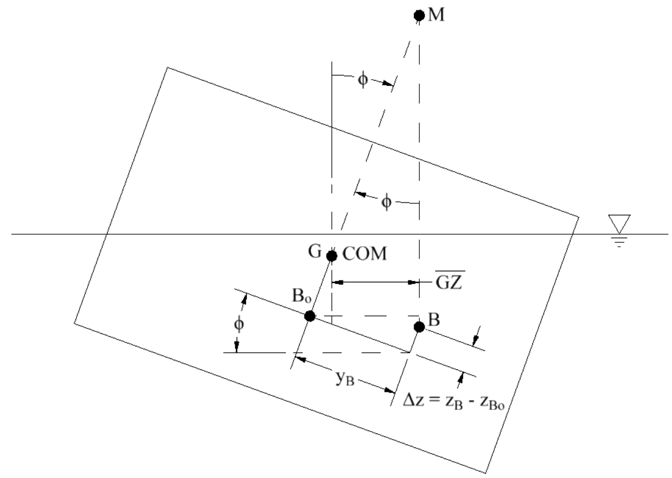

Figure 2: Buoyancy-Gravity Force Couple.

$$\overline{GZ} = \sin(\phi)\left(\overline{GM_x} + \frac{\overline{B_oM_x}}{2}\tan^2(\phi)\right), \tag{7}$$

which is equivalent to

$$\overline{GZ} = y_B\cos(\phi) + (z_B - z_{Bo})\sin(\phi) - \overline{B_oG}\sin(\phi). \tag{8}$$

Note that a positive roll angle ϕ results in a rightward displacement y_B and upward displacement $z_B - z_{Bo}$ in the center of buoyancy, relative to the ship's frame of reference.

The distance between the upright center of buoyancy and the metercenter $\overline{B_oM_x}$ is obtained via the following expression

$$\overline{B_oM_x} = \frac{\int_{A_{wl}} y^2 dA}{V}, \tag{9}$$

where A_{wl} is the area of the ship's waterplane; y is the rightward coordinate on the ship's body fixed frame; and V is the displaced volume of the ship obtained via $\frac{m}{(\rho_w g)}$.

The metacentric height $\overline{GM_x}$ is obtained via the geometric relationship,

$$\overline{GM_x} = \overline{B_oM_x} - \overline{B_oG}, \tag{10}$$

where $\overline{B_oG}$ is the distance between the upright center of buoyancy and the center of mass.

Stability favors a positive righting arm, $\overline{GZ}$, for a positive roll angle. A stability problem arises when a ship's righting arm changes sign from positive to negative at $\phi = 0°$, as this indicates that $\phi = 0°$ is an unstable equilibrium orientation and that there exists a loll angle, an unwanted equilibrium position where $\phi \neq 0°$. Integration of equation 1 with respect to roll angle yields a potential energy curve in which one can distinguish between stable and unstable equilibrium roll angles. With more geometrical information of the vessel, namely the plane of its deck, it is possible to determine the angle of vanishing stability, if one exists [7]. The potential energy curve, illustrating static roll stability for the ship studied in this paper, is featured in Fig. 3. The concave upward portion of the curve indicates that $\phi = 41°$ is a stable equilibrium point, and the two concave downward portions show that $\phi = 0°$ and $\phi = 80°$ are unstable equilibrium points.

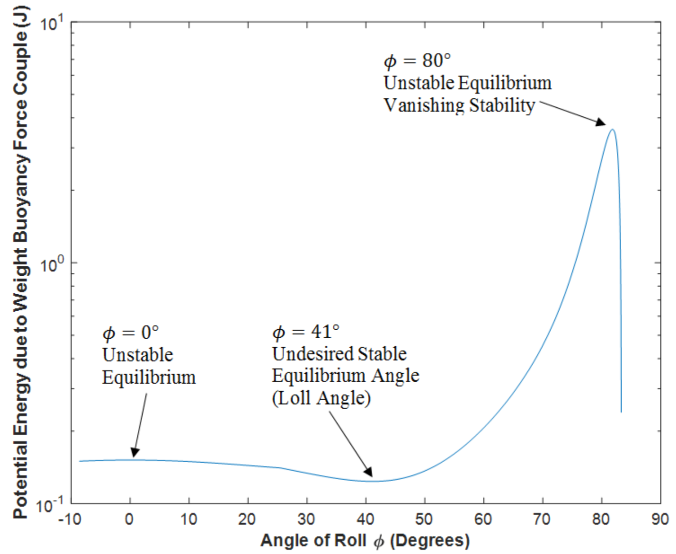

Figure 3: Potential Energy Curve with Respect to Arbitrary Reference Energy Level for the Ship Analyzed in this Paper.

2.3 Proposed Control Law

A control law for the pendulum is proposed to stabilize the ship, based on equilibrium considerations obtained by setting the equations of motion, eq. (4), equal to the zero vector. When f_2 is set to zero, an expression for the equilibrium pendulum angle $\theta_{cw,ref}$ is obtained in terms of the desired equilibrium torque $T_{pend,eq}$ and the current roll angle ϕ,

$$\theta_{cw,ref} = \sin^{-1}\left(\frac{T_{pend,eq}}{m_{cw} g L_{cw}}\right) - \phi. \tag{11}$$

The desired equilibrium torque is obtained from the substitution of f_2 into f_1, yielding

$$T_{pend,eq} = -mg\sin(\phi)\left(\overline{GM_x} + \frac{\overline{B_oM_x}}{2}\tan^2(\phi)\right) + m_{cw} g\left((L_{cw} + L_{off})\sin(\phi)\right). \tag{12}$$

With $\theta_{cw,ref}$ defined, the control law is stated for the servo applied torque on the pendulum T_{pend}:

$$T_{pend} = k_1\left(\theta_{cw,ref} - \theta_{cw}\right). \tag{13}$$

Since perfect equilibrium cannot be achieved by the pendulum servo, a second proportional feedback gain $\tilde{k}_2$ on roll is introduced for stability purposes.

$$T_{pend} = k_1\left(\sin^{-1}\left(\frac{T_{pend,eq}}{m_{cw} g L_{cw}}\right) - \phi - \theta_{cw}\right) + \tilde{k}_2\phi. \tag{14}$$

where $\tilde{k}_2 = k_1 + k_2$ and $k_1 > k_2 > 0$. Eq. (14) can be rewritten as

$$T_{pend} = k_1\left(\sin^{-1}\left(\frac{T_{pend,eq}}{m_{cw} g L_{cw}}\right) - \theta_{cw}\right) + k_2\phi. \tag{15}$$

The control law, eq. (15), consists of a nonlinear feedforward term that establishes an equilibrium point for the boat and a linear feedback term to stabilize the boat about the equilibrium point. Gains k_1 and k_2 are weights for the feedforward and feeback terms, respectively. Knowledge of the roll angle ϕ and θ_{cw} can come from an inertial measurement unit and shaft potentiometer, respectively. In eq. (15), it is possible to fix the angle of $\theta_{cw,ref}$ to a desired angle, such that the controller becomes linear.

This can have benefits for microcontrollers, which have limited storage and computational speed.

For low levels of instability in which the ship does not have a right half plane zero, it can be shown via feedback linearization that the feedback term in eq. (15) is sufficient to stabilize the ship. This approach yields an estimate for the control gain k_2 and is executed as follows. If one were to cancel out all the nonlinear terms in the uncoupled dynamic equations of motion, using the torque input T_{pend}, one obtains an expression that incorporates nonlinear feedback terms in addition to linear feedback terms [9]:

$$\frac{T_{pend}}{m_{cw} g L_{cw}} = \sin(\phi+\theta_{cw})\left(1-\frac{L_{cw}}{(L_{cw}+L_{off})\cos(\theta_{cw})}\right)+ \frac{\sin(\phi)}{\cos(\theta_{cw})}+\frac{m\sin(\phi)\left(\overline{GM_x}+0.5\overline{B_oM_x}\tan^2(\phi)\right)}{m_{cw}(L_{cw}+L_{off})\cos(\theta_{cw})}+ \frac{L_{cw}\dot{\theta}_{cw}^2\tan(\theta_{cw})}{g}-\frac{k_p\phi}{m_{cw} g L_{cw}}, \tag{16}$$

where k_p is a proportional feedback gain on roll angle. For roll angles between $\pm 20^\circ$ with marginal instability and given that $\dot{\theta}_{cw}^2 \approx 0$, small angle approximation may be applied to eq. (16), yielding a purely proportional feedback law,

$$\frac{T_{pend}}{m_{cw} g L_{cw}} = \left(\frac{m\left(\overline{GM_x}+0.25\overline{B_oM_x}\right)}{m_{cw}(L_{cw}+L_{off})}+1-\frac{k_p}{m_{cw} g L_{cw}}\right)\phi. \tag{17}$$

This then can be rewritten as

$$T_{pend} = \left(\frac{m\left(\overline{GM_x}+0.25\overline{B_oM_x}\right) g L_{cw}}{(L_{cw}+L_{off})}+m_{cw} g L_{cw}-k_p\right)\phi. \tag{18}$$

By setting $k_p = 0$ and $k_1 = 0$, comparison of eq. (15) with eq. (18) gives an estimate of the linear feedback gain k_2:

$$k_2 = \left(\frac{m\left(\overline{GM_x}+0.25\overline{B_oM_x}\right) g L_{cw}}{(L_{cw}+L_{off})}+m_{cw} g L_{cw}\right). \tag{19}$$

This results in the following control law:

$$T_{pend} = k_2\phi. \tag{20}$$

Eq. (20) is the minimum required structure of the control law, which is applicable to a marginally stable ship. The estimated feedback gain k_2 is also useful for heuristically estimating the value of the feedforward gain k_1, simply by setting $k_2 = k_1$. Values for these two gains can then be tweaked to attain a desired response. This approach was used to obtain values for gains k_1 and k_2 for the ship discussed in this paper.

2.4 Stability of the Controlled Ship

Lyapunov stability criterion requires that a stable controller should minimize a Lyapunov function $V(\mathbf{x})$. A suitable function for $V(\mathbf{x})$ is the total mechanical energy, E_{mech}:

$$E_{mech} = KE + PE. \tag{21}$$

In order to satisfy the Lyapunov stability criterion, the time derivative of the Lyapunov function, $V(\mathbf{x})$, must be negative semidefinite for all time t [9]. This means that the control law must cause the mechanical energy of the ship-pendulum system to decrease with time for guaranteed stability. The time derivative may be evaluated as follows, using the chain rule:

$$\frac{dV(\mathbf{x})}{dt} = \frac{dV(\mathbf{x})}{d\mathbf{x}}\cdot\dot{\mathbf{x}} \tag{22}$$

Eq.'s (21) and (4) can be substituted into eq. (22), giving

$$\frac{dV(\mathbf{x})}{dt} = \frac{dE_{mech}}{dt} = \frac{dE_{mech}}{d\mathbf{x}}\cdot E^{-1} f(\mathbf{x},\mathbf{u}) = -\frac{\partial L}{\partial t}. \tag{23}$$

Analytically evaluating eq. (23) yields the following expression:

$$\frac{dE_{mech}}{dt} = -b_{roll}\dot{\phi}^2 - b_{pend}\dot{\theta}_{cw}^2 + T_{pend}\dot{\theta}_{cw}. \tag{24}$$

With some algebraic manipulation, one obtains

$$\left(\frac{{T_{pend}}^2}{4b_{pend}}-\frac{dE_{mech}}{dt}\right) = b_{roll}\dot{\phi}^2 + b_{pend}\left(\dot{\theta}_{cw}-\frac{T_{pend}}{2b_{pend}}\right)^2. \tag{25}$$

Since the right hand side of eq. (25) is positive, then

$$\frac{{T_{pend}}^2}{4b_{pend}} \geq \frac{dE_{mech}}{dt}. \tag{26}$$

Given a control law for $T_{pend} = T_{pend}(\phi,\theta_{cw})$, eq. (26) excludes angular velocities from consideration and forms the upper bound for the energy time derivative. It helps visualize the objective of a controller without integrating the equations of motion. Fig. 4 shows the resulting energy derivative bound ${T_{pend}}^2/(4b_{pend})$ for the control law stated in eq. (15).

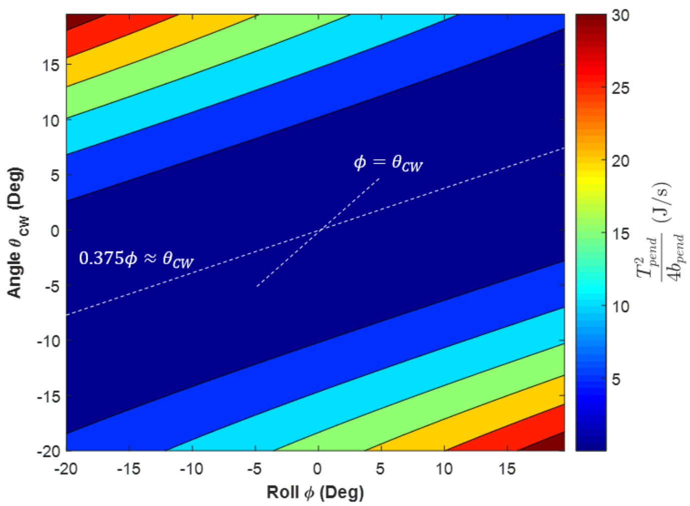

Figure 4: Upper Bound on Mechanical Energy Derivative.

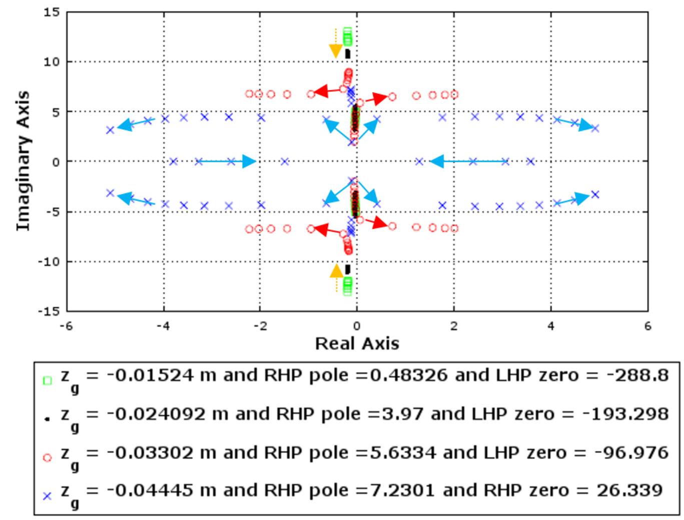

Figure 5: Closed Loop Poles vs. Increasing Roll ϕ for different COM z_g. The sign of z_g is determined by the ship's body fixed coordinate system, depicted in Fig. 10.

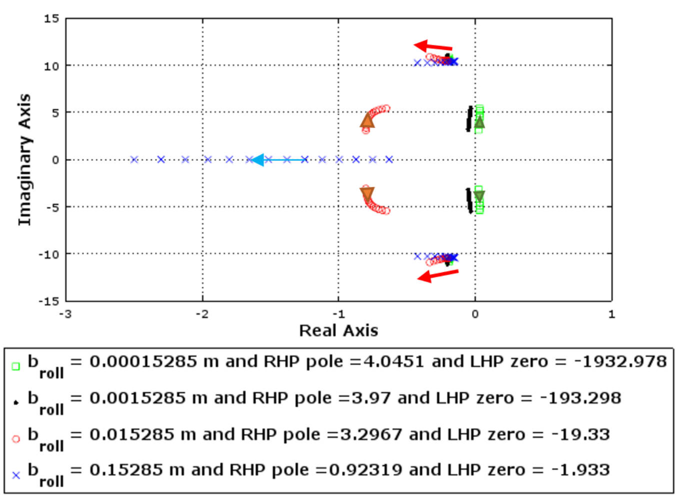

Figure 6: Closed Loop Poles vs. Increasing Roll ϕ for Different Roll Damping Values b_{roll}.

The contour in Fig. 4 characterizes a parabolic cylinder whose values are approximately 0 J/s for small roll angles along the line $\phi_{eq} = \theta_{cw,eq}$, meaning that the derivative of the Lyapunov function, $\frac{dV(\mathbf{x})}{dt}$, is approximately negative semidefinite along this line and that the controller described by eq. (15) should be Lyapunov stable for all equilibrium points near $\phi = 0°$. Intuitively, this result makes sense because a positive pendulum angle will tend to correct a positive roll angle, as shown in Fig. 1. Unfortunately, this method does not take angular velocities and damping into account—so a second stability analysis is needed.

Another means of evaluating closed loop stability about equilibrium points is a root locus plot of the closed loop poles of a state matrix, A, which can be obtained via the decoupling and linearization of eq. (4) about equilibrium roll angles and externally applied torques. The linear state space is expressed as

$$\frac{d\mathbf{x}}{dt} = E^{-1} f(\mathbf{x}, \mathbf{u}) \approx A\delta\mathbf{x} + B\delta\mathbf{u}, \tag{27}$$

where $\delta\mathbf{x}$ and $\delta\mathbf{u}$ are deviations from the equilibrium state and input, respectively. In Figs. 5-6, the plots of the eigenvalues of the state matrix A for different equilibrium roll angles show stability trends for how stability changes with increasing roll damping; increasing vertical center of mass position relative to the water line; and increasing equilibrium roll angle indicated by the direction of the arrows for a range of $\phi = [-20°, 20°]$. These plots show that high roll damping and low center of mass are desired for poles to remain in the left half plane. In other words, stabilization is only possible at marginal stability.

Using small angle approximation and assuming small angular velocities, the open loop RHP pole, p_{ship}, can be estimated as follows:

$$p_{ship} = -\frac{b_{roll}}{2I_{xx}} \pm \frac{1}{2}\sqrt{\left(\frac{b_{roll}}{I_{xx}}\right)^2 - 4a_c}, \tag{28}$$

where a_c is defined as

$$a_c = \left(\left(\frac{mg\overline{GM_x} + m_{cw} g L_{off}}{I_{xx}} \right) + \left(\frac{(L_{off} + L_{cw}) m_{cw} g L_{cw}}{I_{xx} L_{cw}} \right) \right). \tag{29}$$

If the pendulum angle is fixed relative to the ship at $\theta_{cw} = 0°$, then a_c is defined as

$$a_c = \left(\frac{mg\overline{GM_x}}{I_{xx}} \right). \tag{30}$$

Using the same small angle approximation, the open loop RHP zero can be estimated as

$$z_{ship} = \frac{mg}{b_{roll}} \left(-\overline{GM_x} - \frac{m_{cw}}{m} L_{off} - \frac{(I_{xx} + m_{cw}(L_{cw} + L_{off})^2)}{m(L_{cw} + L_{off})^2} \right). \tag{31}$$

In eq. (31), the metacentric height $\overline{GM_x}$ determines the sign of z_{ship}. A negative $\overline{GM_x}$, corresponding to instability, tends to shift the zero towards the RHP. Since the zero is left half plane in the case of marginal stability, care should be taken to ensure that the LHP zero is not too close to the origin, as it would decrease the gain and phase margins, hurting the closed loop response. As the location of the center of mass increases above the waterline beyond the regime of marginal stability, the zero will switch sign, becoming RHP. If both z_{ship} and p_{ship} are RHP, a requirement and limitation on pendulum actuator bandwidth can be stated as long as the RHP is four times greater than the RHP zero. However, z_{ship} can also be less than four times p_{ship} for a range of z_g, as shown in Fig. 5, thus preventing practical feedback control.

2.5 Performance of the Controlled Ship

In the following plots figs. 7-9, two types of responses are compared, disturbance rejection and reference tracking, producing respectively upright ships and tilted ships. For the case of an upright ship featured in fig. 8, the control law (15) is used to stabilize the ship from a nonzero intital roll angle to a zero roll angle—this response is also interpreted as the rejection of an impulse disturbance torque. For the case of the tilted ship, the control system tracks a reference roll angle ϕ_{ref} of 2°, simply by fixing the value of $\theta_{cw,ref}$ obtained from equations (11) and (12) evaluated at ϕ_{ref}. The initial conditions of the tilted ship include a pendulum angle of $\theta_{cw,ref}$ and a roll angle ϕ of 3°. Responses for different damping conditions are recorded to show the sensitivity of roll stabilization to combinations of roll damping b_{roll} and pendulum damping b_{pend}. Mechanical energy plots, normalized by initial mechanical energy, accompany the responses to show whether the controller is Lyapunov stable (i.e. energy dissipating). The values of damping coefficients for different damping conditions are presented in table 1. Estimation of the roll damping coefficient was obtained via the Ikeda Method [2], [10].

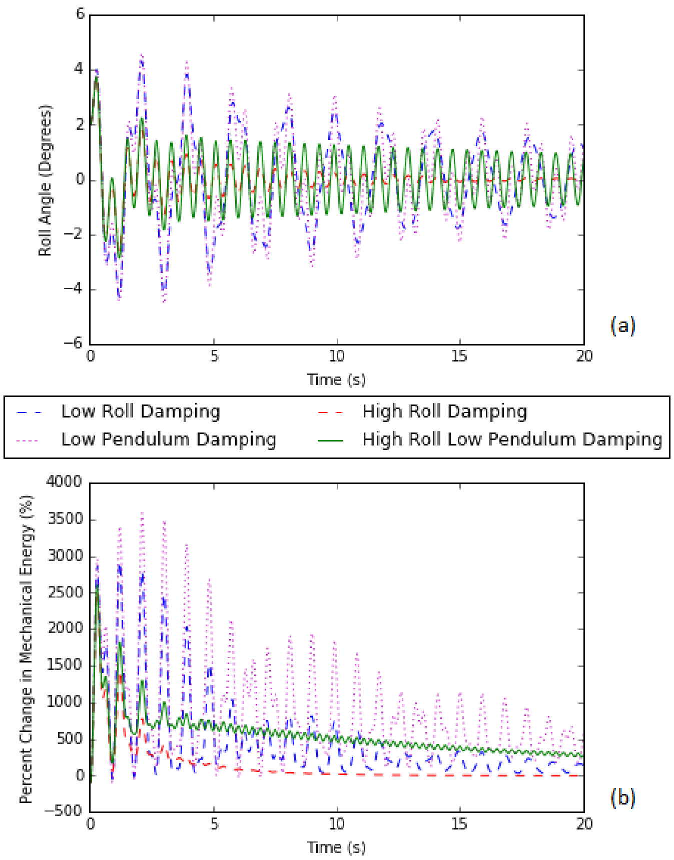

Figure 7: Disturbance Rejection of Ship with Initial Angle of 2°: (a) Roll Response, (b) Percent Change in Mechanical Energy.

3 DISCUSSION OF THE CONTROL SYSTEM

The rate of mechanical energy decay measures the success of a roll stabilization response, as demonstrated in Fig. 7(b) and 8(b). The industry rule of thumb does not change—high roll damping is still desired for a high actuator bandwidth and a high energy dissipation rate. Both cases of high ship damping exhibited favorable responses with an effective exponential energy decay taking place in a 4 second interval. Energy decay is also evidence of Lyapunov stability. In Fig. 9, both roll angle ϕ and pendulum angle θ_{cw} tend be on the line $\phi = \theta_{cw}$ for the reference

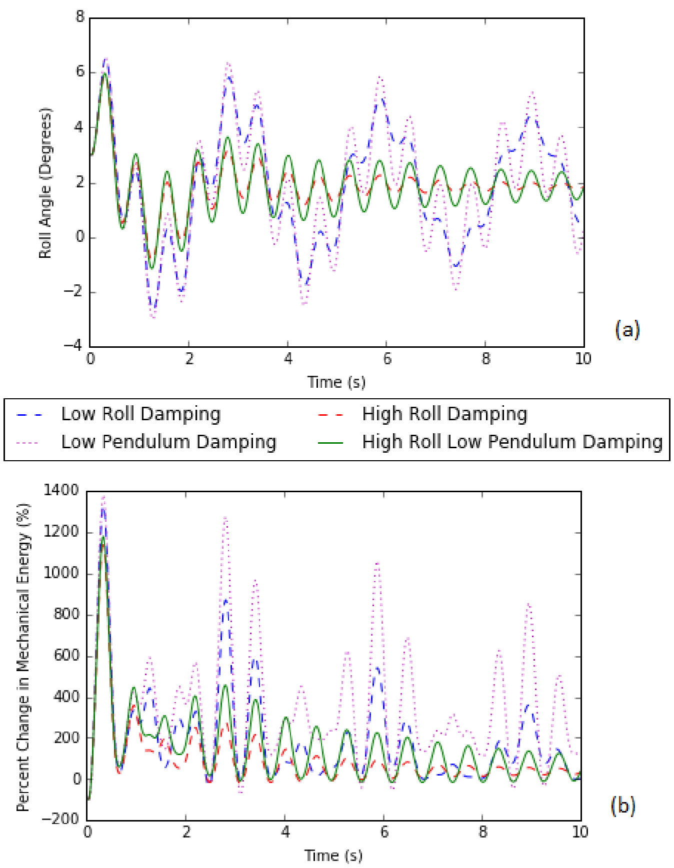

Figure 8: Reference Tracking of 2° for Ship with Initial Angle of 3°: (a) Roll Response, (b) Percent Change in Mechanical Energy.

Damping Conditions	b_{roll} $(N \cdot m \cdot s \cdot rad^{-1})$	b_{pend} $(N \cdot m \cdot s \cdot rad^{-1})$
Low Roll Damping	0.00153	0.001
Low Pend. Damping	0.00153	0.0001
High Roll Damping	0.0153	0.001
High Roll Low Pend. Damping	0.0153	0.0001

Table 1: Table of Damping Coefficient Values

tracking ship, which is maintaining a roll angle of 2°. This energy dissipitating trend is consistent with the energy derivative surface shown in Fig. (4). Pure proportional control was also found to be a satisfactory for the tilted ship. Both linear and non-

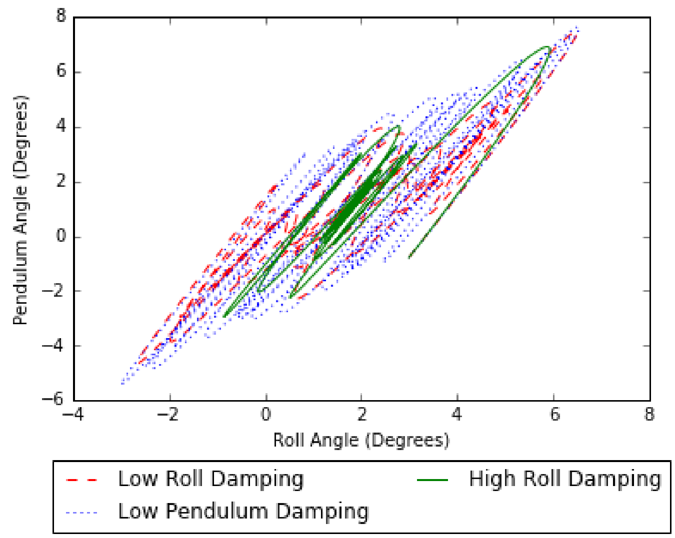

Figure 9: Pendulum Angle vs. Roll Angle of 2° Tilted Ship

linear controllers yielded good responses, but the 2° reference tracking ship displayed superior performance at low roll damping, as demonstrated by the energy decay graphs in Fig.'s 7(b) and 8(b). The major pitfall of the righting ship is that the derivative of the Lyapunov function is negative semidefinite since actuation should be zero at $\phi = 0°$. With the introduction of wave excitation, the righting disturbance rejecting ship may move in and out of stability regimes at $\phi = 0°$ [4]. For the sake of practically, this region should be avoided, especially if the ship has insufficient roll damping—hence, the ship should be stabilized at a roll angle greater than zero, so that the ships roll angle changes sign much less frequently. This tilt angle could be 1.5° to 2°. From Fig.'s 7(b) and 8(b), one can see that a tilted ship, maintaining an angle of 2°, stores three times less mechanical energy at peak value than the upright disturbance rejecting ship.

4 CONCLUSIONS

The pendulum was able to stabilize a marginally stable ship at a tilted roll angle and a zero roll angle, thus demonstrating its value as a benchmark solution to the problem of marginal ship stability. A linear controller was found to be suitable for roll angles less than 20°. Even though wave excitation was not considered, one should see the merit of using impulse signals (IC's) in simulating ship stabilization responses because, in the linear regime of ±10°, a response to an arbitrary disturbance torque signal can be recreated from the linear superposition of multiple impulse responses. From the energy decay plots, it was found that a tilted reference tracking ship produced a more favorable mechanical energy decay than the upright disturbance rejection ship. The major benefit behind tracking the tilted roll angle was that it prevented the ship from oscillating dangerously about $\phi = 0°$. A tilted ship was less prone to overshoot. A loll angle, as shown in Fig. 3, would do quite the opposite for the uncontrolled case, which is why it is dangerous [7]. Finally, roll damping was the second most critical feature in the roll stabilization of a marginally stable ship. Roll damping should be at least the same magnitude as pendulum damping in order to give the pendulum actuator sufficient time to react. Yet even with high roll damping, the control system should minimize the occurence of the ship swinging past $\phi = 0°$ in either direction. Further work is being done in validating the conclusions of this paper with experiment, as well as finding more compact ways of implementing the pendulum onboard an actual ship. Fig. 10 shows one possible configuration for installation. Additionally, simulations of a 7-DOF ship-pendulum system are being run to simulate the effects of maneuvering, motion coupling, and wavy seas on roll stabilization. By analyzing this coupling, one can determine whether a stabilized, marginally stable ship can safely conduct maneuvers and navigate.

ACKNOWLEDGMENT

We thank the sponsorship of the Department of Mechanical Engineering at the Cooper Union for the Advancement of Science and Art.

REFERENCES

[1] Chen, S. L., Shaw, S. W., Khalil, H. K., and Troesch, A. W., 2000. "Robust stabilization of large amplitude ship rolling in beam seas". *J Dyn Syst-T ASME,* ***122***, pp. 108–113.

[2] Pesman, E., Bayraktar, D., and Taylan, M., 2007. "Influence of damping on the roll motion of ships". In The 2nd International Conference on Marine Research and Transportation, pp. 127–133.

[3] Kim, H., Haugen, A., and Utne, I. B., 2016. "Assessment of accident theories for major accidents focusing on the MV SEWOL disaster: Similarities, differences, and discussion for a combined approach". *Safety Science, Elsevier,* ***82***, pp. 410–420.

[4] Dimentberg, M. F., Hera, A., and Naess, A., 2008. "Marginal Instability and Intermittency in Stochastic Systems—Part I: Systems with Slow Random Variations of Parameters". *J Appl Mech,* ***75***, pp. 1–8.

[5] Skogestad, S., and Postlethwaite, 2010. *Multivariable Feedback Control: Analysis and Design, 2nd Ed.* John Wiley and Sons Inc., Chichester, England, UK.

[6] Papoulias, F. A., and McKinley, B. D., 1994. "Inverted Pendulum Stabilization of Submarines in Free Positive Buoyancy Assent". *SNAME,* ***38(1)***, pp. 71–82.

[7] Tupper, E. C., 2004. *Introduction to Naval Architecture, 4th Ed.* Waltham, MA, USA. Chap. 7.

[8] Lee, S., 2000. "Analysis of an initially inclined ship in following sea". *J Appl Mech,* ***67***, pp. 717–719.

[9] Khalil, H. K., 2001. *Nonlinear Systems*. Pearson, Chap. 4 and 13.

[10] Himeno, Y., 1981. *Prediction of Roll Damping—State of the Art*. University of Michigin, College of Engineering.

Appendix A: 7-DOF Model

The following lagrangian describes a 7-DOF model of the ship and pendulum.

$$
\begin{aligned}
L = {} & \frac{1}{2}\left[p\ q\ r\right]\left[I_{ij}\right]\begin{bmatrix}p\\q\\r\end{bmatrix} + \frac{1}{2}\left[u\ v\ w\right]\begin{bmatrix}m&0&0\\0&m&0\\0&0&m\end{bmatrix}\begin{bmatrix}u\\v\\w\end{bmatrix} + \\
& \left(\vec{r}_{com}\times m\left[u\ v\ w\right]^T\right)^T\left[p\ q\ r\right]^T + \\
& + \frac{m_{cw}\dot{\theta}_{cw}^2 {L_{cw}}^2}{2} + \frac{m_{cw}(L_{cw}+L_{off})^2(p^2+q^2)}{2} + \\
& - m_{cw}uq(L_{cw}+L_{off}) + m_{cw}vp(L_{cw}+L_{off}) + \\
& - m_{cw}v\dot{\theta}_{cw}L_{cw}\cos(\theta_{cw}) - m_{cw}w\dot{\theta}_{cw}L_{cw}\sin(\theta_{cw}) + \\
& - m_{cw}p\dot{\theta}_{cw}L_{cw}(L_{cw}+L_{off})\cos(\theta_{cw}) + \\
& - mg\left(\overline{GM_x}(1-\cos(\phi)) - 0.5\overline{B_oM_x}(2-\sec(\phi)-\cos(\phi))\right) + \\
& - mg\left(\overline{GM_y}(1-\cos(\theta)) - 0.5\overline{B_oM_y}(2-\sec(\theta)-\cos(\theta))\right) + \\
& + m_{cw}g\left((L_{cw}+L_{off})\cos(\phi) - L_{cw}\cos(\phi+\theta_{cw})\right) + \\
& + (mg-B_o)z_{global} - \frac{1}{2}\rho_w g A_{plan} {z_{global}}^2
\end{aligned} \tag{32}
$$

Nonconservative forces may be added after differentiation of eq. (32).

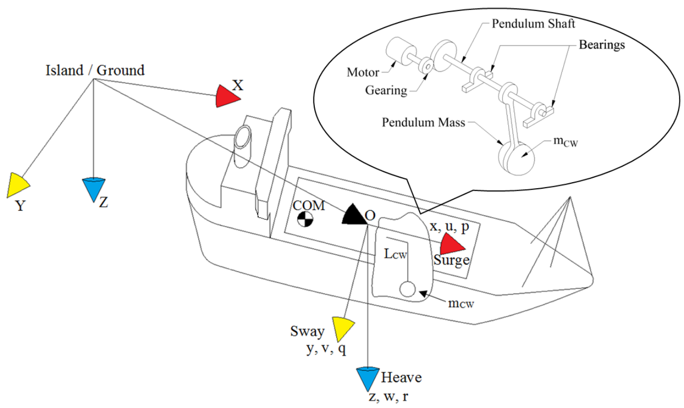

Figure 10: Sketch of 7-DOF Ship with Pendulum

Appendix B: Numerical Values of the Ship Studied in this Paper

m	$1.681kg$
I_{xx}	$0.009058kg \cdot m^2$
m_{cw}	$0.125kg$
g	$9.81m \cdot s^-2$
b_{roll}	$0.0015285N \cdot m \cdot s \cdot rad^-1$ as base value
b_{pend}	$0.001N \cdot m \cdot s \cdot rad^-1$ as base value
z_g	$-0.0241m$, which is above the waterline at $\phi = 0°$
ρ_w	$998kg \cdot m^-3$
$\forall$	$0.00168m^3$
$\overline{B_oG}$	$0.05013m$
$\overline{B_oM_x}$	$0.04111m$
L_{cw}	$0.2m$
L_{off}	$0.0m$
k_1	0.3
k_2	0.3
k_p	0.265
z_{ship}	-193.3
p_{ship}	3.9700